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Couette Flow of a Bingham Fluid in a Channel Bounded By Permeable Beds with Suction and Injection

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Abstracts
Viscous flow through and past porous media is of fundamental importance in various branches of Science and Technology. Fluid flows involving Porous media occurs in oil production, filtration, coalescence and in biological transport. The petroleum engineers are interested in seepage problems arising in extraction of petroleum from oil wells. The civil engineers and geologists are interested to know the ground water movement in porous layer in earth in order to tap water effectively and solve water scarcity problems in drought areas. The biologists are interested in water movement through plants which needs the knowledge of flow through porous media. Hence it is necessary to study flow through and past porous media. Some recent studies in this direction are given in references [19-25].

Keywords: Channel Bounded.

Introduction

Viscous flow through and past porous media is of fundamental importance in various branches of Science and Technology. Fluid flows involving Porous media occurs in oil production, filtration, coalescence and in biological transport. The petroleum engineers are interested in seepage problems arising in extraction of petroleum from oil wells. The civil engineers and geologists are interested to know the ground water movement in porous layer in earth in order to tap water effectively and solve water scarcity problems in drought areas. The biologists are interested in water movement through plants which needs the knowledge of flow through porous media. Hence it is necessary to study flow through and past porous media. Some recent studies in this direction are given in references [19-25].

Berman [1] studied the problem of 2D steady-state Newtonian laminar flow in a channel with Porous walls. Bird et al. [2] investigated the Bingham fluid flow in a rigid circular tube. Brady [4] reported that for tubes that are very long compared to their radius, the inlet velocity profile for flows above critical Reynolds number does not necessarily decay in to the fully developed similarity profile for an infinite tube. In view of several applications in biomechanics, Vajravelu et al. [5] made a study on the Bingham fluid flow in a circular tube with permeable wall. The velocity field is obtained using Beavers and

Joseph [3] slip condition at the permeable wall. The Bingham fluid flow between two permeable beds is discussed by Govardhan et. al. [6]. Comparini [7] discussed a one-dimensional model for the time dependent flow of a Bingham fluid between two parallel plates.

Couette flow of Bingham Plastics was considered by Loest et al. [8] who performed two dimensional numerical simulations of Bingham slurries for the forming flow of ceramic tapes using finite element methods. Ravana et al. [9] studied the free surface flow of a Bingham fluid in an inclined channel over a permeable bed. The flow in the channel is described by Bingham model, where as the flow in the permeable bed is according to Darcy law. The velocity field, the shear stress, the mass flow rate and its fractional increase are obtained. Narahari et al. [10] discussed unsteady flow of a Bingham fluid between two permeable beds having different permeabilities. The velocity distribution in the porous and non-porous regions are obtained. Sankara Reddy et al. [11] made a detailed study on the Bingham fluid flow in an inclined channel bounded by two permeable beds. The velocity distributions in the porous and non-porous regions are obtained. Petrov [12] examined analytically the unsteady flow of Bingham fluid caused by an abruptly applied pressure gradient. The work by Karode [13] deals with analytical solutions of pressure drop in Newtonian flows in a tube and a rectangular slit with porous walls.

Ariel [14] obtained exact analytical solutions of laminar flow of a second grade visco elastic fluid employing two geometries. Oxarango et al. [15] to introduce a one-dimensional model so as to describe the laminar Newtonian fluid flow in a two dimensional or circular porous channel with wall suction or injection. Heat transfer was also taken in to account in the solution of the incompressible. Newtonian Couette flow with porous plates in the work by Fang [16]. Kamisli [17] discussed the laminar Couette flow of Non-Newtonian fluid in channels with wall suction/ injection using asymptotic techniques. Sokrates Tsangaris et al. [18] analyzed Couette flow of a Bingham plastic in a channel with equally porous parallel walls.

In this chapter, the Couette flow of a Bingham fluid between two parallel permeable beds with suction and injection is investigated. The upper bed moves with constant velocity V and the lower bed is kept stationary. The velocity field in the plug flow and non-plug flow regions is obtained. The effect of three dimensionless parameters viz. Bingham, Couette and Reynolds numbers on the velocity is discussed.

Nomenclature

x, y   Cartesian co-ordinates
u   Velocity component in x-direction
k_1 Permeability of the lower bed
k_2 Permeability of the upper bed
Q_1 Darcy velocity in the lower bed
Q_2 Darcy velocity in the upper bed
h   Width of the channel
α Slip parameter
u_{B1} Slip velocity at y =0
u_{B2} Slip velocity at y=h
p   Pressure
ρ Density of the fluid
μ Coefficient of viscosity
τ_0 Yield stress of the fluid
τ Local shear stress
u_1 Velocity component in $0 \leq y \leq h_1$
u_2 Velocity component in $h_1 \leq y \leq h_2$
u_3 Velocity component in $h_2 \leq y \leq 1$

Re   Transverse Reynolds number, $\left(\dfrac{V \rho h}{\mu} \right)$

Bn   Bingham number, $\left(\dfrac{\tau_h}{\mu U_P} \right)$

Co   Couette number, $\left(\dfrac{U}{U_P} \right)$

V   Suction/ Injection velocity

Mathematical formulation of the problem

Consider the Couette flow of a Bingham fluid between two permeable beds as shown in Fig.1. The permeabilities of lower and upper beds are k_1 and k_2 respectively. The lower bed is stationary and the upper bed moves with constant velocity u. The fluid is injected into the channel from the lower bed with a velocity V and is sucked out in to the upper bed with the same velocity V. The width of the channel is h. X-axis is taken along the nominal surface of the permeable bed and y-axis is taken perpendicular to x-axis. The flow between the two permeable beds is governed by Bingham Model and the flow in the permeable beds is according to Darcy's law. The flow is driven by a constant pressure gradient. Cartesian co-ordinate system is used.
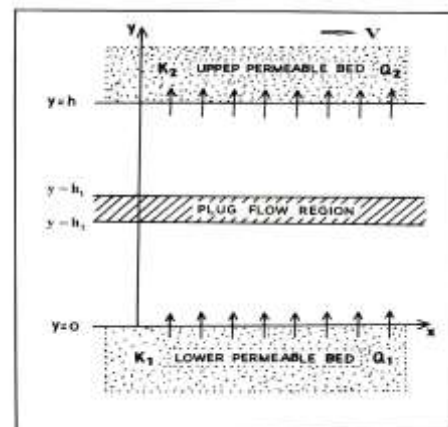

Fig.1: Physical Model

Basic equations

$$\rho v \frac{du}{dy} = -\frac{dp}{dx} + \frac{d\tau}{dy} \qquad (3.1)$$

$$\tau = \mu \frac{du}{dy} + \tau_y \ \text{Sign} \left(\frac{du}{dy} \right), |\tau| > \tau_y \quad (3.2)$$

$$\frac{du}{dy} = 0, |\tau| < \tau_y \qquad (3.3)$$

Boundary conditions

$$u_1 = u_{B_1} \qquad \text{at} \quad y = 0 \qquad (3.4)$$

$$u_2 = u_{B_2} + Co \quad \text{at} \quad y = h_2 \qquad (3.5)$$

$$\left(\frac{du_1}{dy}\right)_{y=h_1} = \left(\frac{du_2}{dy}\right)_{y=h_2} = 0 \qquad (3.6)$$

$$u_1(h_1) = u_2(h_2) \qquad (3.7)$$

$$u_1 = u_2 \text{ at } y = h_1 \qquad (3.8)$$

$$u_2 = u_3 \text{ at } y = h_2 \qquad (3.9)$$

$$\left(\frac{du_1}{dy}\right)_{y=h_1} = \left(\frac{du_3}{dy}\right)_{y=h_2} = 0 \qquad (3.10)$$

Non- dimensionalization of the flow quantities

The following non-dimensional quantities are introduced to make the basic equations and the boundary conditions dimension less:

$$\bar{y} = y/h \;\; ; \;\; \bar{u} = \frac{u}{U_p}; \quad \bar{\tau} = \frac{\tau h}{\mu U_p};$$

$$U_P = \frac{-h^2}{\mu}\frac{dp}{dx}; \quad Re = \frac{V\rho h}{\mu};$$

$$c_0 = \frac{U}{U_P}, B_n = \frac{\tau_y h}{\mu U_p}$$

In view of the above dimensionless quantities equations $(3.1) - (3.3)$ reduces to the following form. The bars (-) are neglected here after.

$$\frac{du}{dy} - Re\, u = -y - Bn \; \text{Sign}\left(\frac{du}{dy}\right) - k,$$

$$|\tau| > Bn \qquad (4.1)$$

$$\frac{du}{dy} = 0, |\tau| < Bn \qquad (4.2)$$

$$u = u_{B_1}, \frac{du}{dy} = \beta_1\left(u_{B_1} - Q_1\right) \text{ at } y = 0; \qquad (4.3)$$

$$\text{where} \quad \beta_1 = \frac{\alpha}{\sqrt{D_{a1}}}$$

$$u = u_{B2} + c_0; \; \left(\frac{du}{dy}\right) = \beta_2\left(u_{B_2} - Q_2\right) \text{ at } y = 1;$$

(4.4)

$$\text{where} \quad \beta_2 = -\frac{\alpha}{\sqrt{Da_2}}$$

$$\left(\frac{du}{dy}\right)_{y=h_1} = \left(\frac{du}{dy}\right)_{y=h_2} = 0 \qquad (4.5)$$

$$u(h_1) = u(h_2) \qquad (4.6)$$

Solution of the problem

Solving equation (4.1) subject to the boundary conditions (4.3) to (4.6) we get the velocity field as

$$u_1(y) = \frac{1}{Re^2}(Re\,y + 1) + \frac{Bn + k}{Re} + k_1\, e^{Re\,y}$$

$$(0 \le y \le h_1) \qquad (5.1)$$

$$u_2(y) = \frac{1}{Re^2}(Re\,y + 1) + \frac{-Bn + k}{Re} + k_1\, e^{Re\,y}$$

$$(h_2 \le y \le 1) \qquad (5.2)$$

where

$$k_1 = \frac{a\; e^{Re\,h_2}}{e^{Re\,h_2} - e^{Re(1+h_1)}}; \; k_2 = \frac{a\; e^{Re\,h_1}}{e^{Re\,h_2} - e^{Re(1+h_1)}}$$

$$a = \frac{1}{Re} - \frac{2Bn}{Re} - \left(Co + u_{B_2} - u_{B_1}\right)$$

The slip velocity at the lower and upper beds are obtained as

$$u_{B_1} = \frac{A_2 G_2 + A_4 G_1}{-(A_1 A_4 + A_2 A_3)} \qquad (5.3)$$

$$u_{B_2} = \frac{A_3 G_1 - A_1 G_2}{-(A_1 A_4 + A_2 A_3)} \qquad (5.4)$$

where

$$A_1 = d\beta_1 - Re\; e^{Re\,h_2}, \qquad A_2 = Re\; e^{Re\,h_2},$$

$$G_1 = Re\; e^{Re\,h_2} Co - \frac{d}{Re} - 1 + 2Bn - d\beta_1 Q_1$$

$$A_3 = Re\, e^{Re(1+h_1)}, \; A_4 = Re\, e^{Re(1+h_1)} + d\beta_2$$

$$G_2 = \frac{d}{Re} + e^{Re(1+h_1)} - 2e^{Re(1+h_1)}Bn$$

$$\qquad - Re\, e^{Re(1+h_1)}Co + d\;\beta_2\; Q_2$$

$$h_1 = \frac{1}{R_e} \log \frac{e^{Re(1-2Bn)} - 1}{R_e\left(1 - 2Bn - CoR_e\right)},$$

$$h_2 = h_1 + 2Bn$$

Results and discussions

From equations (5.1) – (5.2) we have calculated the velocity u, as a function of y. Velocity profiles are plotted for different values of Brinkman number Bn, with fixed R =5, Co=0.02, $\alpha = 5$, Da = 0.001, and is shown in figure 2. We observe that the velocity increases with increase in y initially and maintains a constant velocity for some length and then it decreases with the increment in y. For a given y, we notice that the velocity decreases with increasing Brinkman number.

Figure 3 shows the effect of Couette number Co on the velocity. We observe that the velocity increases with increase in y initially and maintains a constant velocity for some length and then it decreases with the increment in y. For a given y, the velocity increases with increasing Couette number.

The variation of velocity u for different values of Reynolds number Re is drawn in figure 4. We observe that the velocity increases with increase in y initially and maintains a constant velocity for some length and then it decreases with the increment in y. For a given y, the velocity increases with increasing Reynolds number.
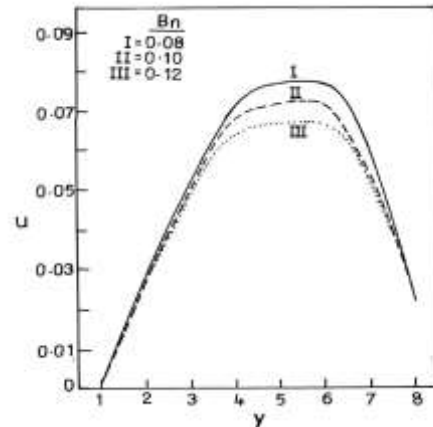
Figure 5 is plotted to observe the effect of different values of Darcy number Da on the velocity u. we found that the velocity increases with increase in y initially and maintains a constant velocity for some length and then it decreases with the increment in y. For a given y, the velocity increases with increasing Darcy number.

Fig.2: **Velocity Profiles for different values of** B_n **with fixed values of** $R = 5, Co = 0.02,$ $Da = 0.001, \alpha = 5$
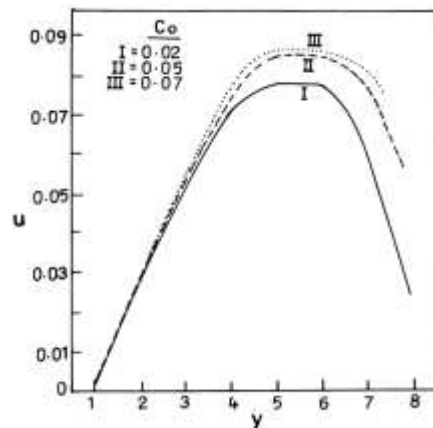

Fig.3: **Velocity Profiles for different values of Co with fixed values of** $R = 5, Bn = 0.08,$ $Da = 0.001, \alpha = 5$
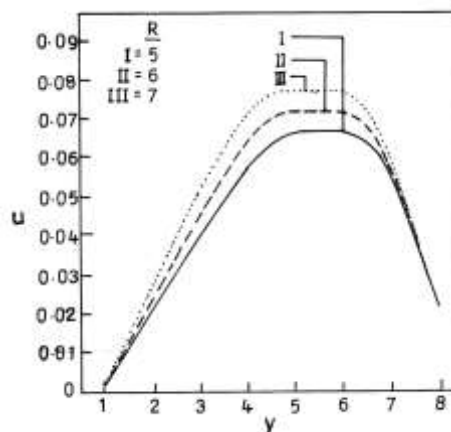
Fig.4: **Velocity Profiles for different values**

of R with fixed values of $Co = 0.02,$
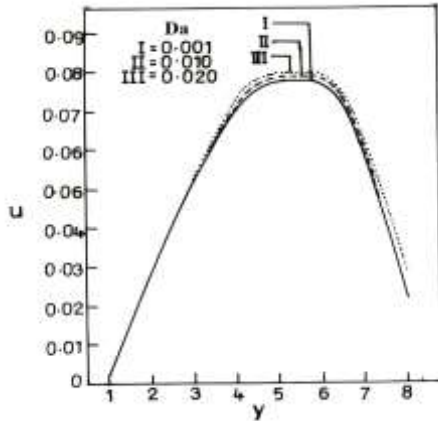
$Bn = 0.08, Da = 0.001, \alpha = 5$

Fig.5: **Velocity Profiles for different values**

of Da with fixed values of $Co = 0.02,$

$Bn = 0.08, R = 5, \alpha = 5$

References

1. Berman A.S., Laminar flow in an annulus with porous walls, J. Appl. Phys., 29 (1958) 71-75.
2. Bird, R. B., Stewart, W. E., and E. N. Lightfoot. Transport Phenomena, John Wiley & Sons, New York (1960).
3. Beavers G.S. and Joseph D.D., Boundary conditions at a naturally permeable wall, J. Fluid Mech., 30 (1967) 197-207.
4. Brady J.F., Flow development in a porous channel and a tube, J. Phys. Fluids, 27 (1984) 1061-1067.
5. Vajravelu K., Sreenadh S., Ramakrishna S. and Arunachalam P.V., Bingham fluid flow through a circular pipe with permeable wall, ZAMM., 67 (1987) 568-569.
6. Govardhan S Sreenadh S. and Arunachalam P.V., Bingham fluid flow between two permeable beds, Proc. Nat. Acad. Sci. India., 61A (1991) 19-27.
7. Comparini E., A one-dimensional Bingham flow, Journal of Mathematical Analysis and Applications, 169 (1992) 127-139.
8. Hagen Loest, Roland Lipp and Evan Mitsoulis, Numerical Flow Simulation of Viscoplastic Slurries and Design Criteria for a Tape Casting Unit, Journal of the American Ceramic Society, 77 (1), (1994) 254–262.
9. Ravana G., Ramakrishna S., Sreenadh S. and Arunachalam P.V., Bingham fluid flow through an inclined channel over a permeable bed, Bulletin of Pure and Applied Sciences. 15E (1), (1996) 47-60.
10. Narahari M., Sreenadh S. and Arunachalam P.V., Unsteady flow of a Bingham fluid between two permeable beds, J. Indian Acad. Math., 22 (2), (1999) 225-242.
11. Petrov A.G.,The development of the flow of viscous and visco elastic Media between two parallel plates. Journal of Appl. Mathematics Mech. 64 (2000) 123-132.
12. Sankara Reddy P., Sreenadh S. and Arunachalam P.V., Bingham fluid flow in an inclined channel bounded by two permeable beds,J. Indian Acad. Math. 22 (1), (2000) 143-162.
13. Karode S.K., Laminar flow in channels with porous walls, revisited, J. Membrane Science, 191 (2001) 237-241.
14. Ariel P.D., On exact solutions of flow problems of a second grade fluid through two parallel porous walls, Int. J. Eng. Sci., 40 (2002) 913-941.
15. Oxarango L., Schmitz P. and Quintard M., Laminar flow in channels with wall suction, or injection: a new mode to study multi channel filtration systems, Chem. Eng. Sci., 59 (2004) 1039-1051.
16. Fang T., A note on the incompressible Couette flow with porous walls, lat. Commun. Heat Mass Transfer 31 (2004) 31-45.
17. Kamisli F., Laminar flow of a non-Newtonian fluid in channels with wall suction or injection, Int. J. Eng. Sci., 44 (2006) 650-661.
18. Sokrates Tsangaris, Christos Nikas, Grigorios Tsangaris and Panagiotis Neofytou, Couette flow of a Bingham plastic in a channel with equally porous parallel walls, J. Non-Newtonian Fluid Mech.,144 (2007) 42-48.
19. Vajravelu K., Sreenadh S. and Lakshminarayana P., The influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Commun. Non-linear Sci. Numer. Simulat., 16 (2011) 3107-3125.
20. Sreenadh S., Lakshminarayana P. and Sucharitha G., Peristaltic flow of micropolar fluid in an asymmetric channel with

permeable walls. Int. J. of Appl. Math. and Mech., 7 (20) (2011) 18-37.

21. Sucharitha G., Sreenadh S. and Lakshminarayana P., Non- linear peristaltic transport of a conducting Prandtl fluid in a porous asymmetric channel, Int. J. of Eng. Research & Technology. 1(10), (2012) 1-10.

22. Nandagopal K, Sreenadh S., Chakradhar K. Vijaya Kumar A.G., Pulsatile flow of a viscous fluid in an inclined channel bounded by two Permeable beds. Far east journal of applied mathematics,(FJAM) 72 (2), (2012) 129-148.

23. Nandagopal K, S. Sreenadh, A.G.Vijaya Kumar., Flow of a couple stress fluid through a porous layer bounded by parallel plates, International Journal of Engineering Research and Technology, 1 (10), (2012).

24. Sucharitha G, Sreenadh S, Lakshminarayana P, Non-linear peristaltic flow of pseudoplastic fluid in an asymmetric channel with porous medium. Int. J. of Eng. Sci. and Tech., 5(1), (2013) 106-113.

25. Avinash K. J.AnandaRao, Y.V.K. Ravikumar and S.Sreenadh., Bingham fluid flow through a Tapered tube with Permeable wall. Journal of Applied Fluid Mechanics, 6 (1), (2013) 143-148